Learning and Teaching Engineering Mathematics within an Active Learning Paradigm.

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ABSTRACT

The purpose of this paper is to report upon how using open-ended, ill-formed problems were used as a capstone project within a level 4 mathematics module to enhance students' higher order thinking skills and complement the competencies they develop through an active learning model. Specifically, it provided students with the opportunities to think mathematically, reason mathematically, pose and resolve mathematical problems, to use technology to model resolutions, interpret and handle mathematical symbolism and to communicate their resolutions to peers and staff.

The evidence from this investigation concludes that the majority of students found the experience challenging but worthwhile. They considered they had learnt important skills including the ability to form assumptions, persistence, time management, project management and enhancement of their mathematical skills in relation to engineering. Many students also thought it was a useful experience in their development as professional engineers.

KEYWORDS

Problem resolution, modelling, analysis, evaluation, synthesis, Standards:

BACKGROUND

An Active Learning paradigm has been at the heart of Mechanical Engineering and Design programmes for many years at Aston University.

Table 1, First-year Mechanical Engineering and Design programme.

YEAR 1	
Teaching Period 1	Teaching Period 2
ME1601 Engineering Science - 20 credits	
Forces and forces in structures, free body diagrams, Statics, mechanics, dynamics - plane and curvilinear motion	Basic fluids, Bernoulli, thermodynamics, heat transfer, heat and power cycles
ME1600 Electronic Engineering Fundamentals - 20 credits	
DC, charge, Kirchoff, Ohm, capacitance, op amps.	timers, binary, hex, microcontrollers
ME11EM Engineering Mathematics - 20 credits	
Arithmetic, eqn of line, logs, trig, complex no., vectors, calculus, introduction to MATLAB	Data, matrices, 1st order ODE, 2nd order ODE, fourier, vector calculus, MATLAB challenges
ME1501 Design and Experimentation - 30 credits	ME1502 Prototyping and Development - 30 credits
Design History File, creativity, erg & anthropometrics, package drawings, stress and design for strength, engineering drawings & geometrical tolerances	Wind energy, efficiency, power curves, user spec, PDS, aerodynamics, structural analysis, sankey & LCA, FMEA - CAD and Excel

Table 1 shows an overview of how the first year of the programme was designed. The programme comprises a series of major practical project modules which were developed in order to afford students the opportunity to learn engineering skills and knowledge of engineering processes in a more practical, experiential manner than the ‘traditional’ lecture approach. It was also recognised that, due to the nature of the students, underpinning theoretical concepts still needed to be taught. Due to this consideration, two theory modules, engineering science and engineering mathematics, were taught separately within the first year of the programme. The outcome from this approach was that students seemed to ‘compartmentalise’ their knowledge and skills i.e. the mathematics knowledge and associated skills they acquired were not necessarily transferred to other modules. Also, in many cases, students questioned the need to learn mathematics and its appropriateness for engineering. In order to ameliorate this situation, it was decided to align the engineering mathematics module closer to the approach taken in the practical project modules (Table 2). In other words, adopt a more active learning approach to the delivery of engineering mathematics. This alignment also resonates with the core philosophy of the CDIO syllabus. The CDIO syllabus states ‘…the three modes of thought most practiced by professional engineers are explicitly called out: Engineering Reasoning and Problem Solving, Experimentation and Knowledge Discovery, and System Thinking’ (Crawley, 2001, p. 5). This has parallels with the Engineering Habits of Mind (EHoM) philosophy advocated by the Royal Academy of Engineering (Lucas, Hanson, & Glaxton, 2014). This report defines the EHoM as: Problem finding, creative problem solving, visualising, improving, systems thinking and adapting.

Table 2, Alignment of Design and Experimentation and Engineering Mathematics.

Stage	ME1501 Design and Experimentation – car project	ME11EM Engineering Mathematics – Matlab challenge
Conceive	Establish project scope and boundary. Specification for knowns. Assumptions for unknowns. Team dynamics. Project Management.	Establish project scope and boundary. Specification for knowns. Assumptions for unknowns. Team dynamics. Project Management.
Design	Apply appropriate mathematics and engineering science. Make design decisions. Develop prototype model.	Identify and apply appropriate mathematics. Make design decisions. Develop initial resolutions and produce Matlab models. Correct errors. Select final solution.
Implement	Produce engineering drawings. Produce manufacturing specifications.	Produce final solution.
Operate	Build and test.	Run Matlab model.

The implementation of ‘engineering thinking’ in a mainly practical module such as the Experimentation and Design, is reasonably straightforward but by no means unproblematic. Translating this approach to a theory based module such as engineering mathematics is challenging.

The report produced by the European Society for Engineering Education (SEFI) Mathematics Working Group (2013) was used as a starting point to identify the skills required by a contemporary engineer in terms of mathematical competencies. This report identified the key competencies as: thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modelling mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating in, with, and about mathematics and, making use of aids and tools (p 14). This set of competencies enabled the process of developing an approach to ‘engineering thinking’ to proceed.

In order to start the process of acquiring these skills within engineering mathematics, it was decided to expose the students to open-ended, ill-formed problems (see Appendix 1 for an example). The students involved with this investigation were first-year undergraduate students studying Mechanical Engineering at Aston University.

The module was organised into three elements: semester one comprised of traditional lectures, tutorials and guided, self-directed learning on Matlab. Semester two took the form of the continuation of traditional lectures, tutorials and a problem resolution session.

For the problem (termed a challenge) resolution session, the students were organised into teams and told to select a challenge from a range of available briefs. Once they had selected a challenge they were expected to find and use appropriate mathematical formulae and procedures in order to develop an abstract model using Matlab. They were given talks on problem-solving and about working in teams. The assessment took the form of an academic poster which would be assessed by members of the Mechanical Engineering and Design faculty. Prior to the assessment day, the students were given instruction on how to design an academic poster. The staff members supervising the sessions were advised not to intervene at an early stage and let the teams struggle with the challenge.

LITERATURE REVIEW

Fundamental to engineering is the necessity for the engineer to be able to analyse a problem, identify the mathematics required to translate the physical scenario into an abstract model, interpret the results of the modelling process and communicate the resolution to managers and peers. The skills implicit within this process are known as higher order thinking skills and were identified in Bloom’s Taxonomy (Forehand, 2005) as analysis, evaluation and synthesis. These higher order thinking skills are difficult to teach and for students to learn but are vital if the complex world of engineering is to advance. According to (Sazsin, 1998, p. 146) ‘most engineering students think in terms of numbers rather than in terms of abstract concepts’. This ‘attitude’ towards mathematics tends to be encouraged at school or college where mathematics can be taught as a series of procedures analogous to a machine where one enters the inputs, performs a process which generates an output. At the other end of the spectrum, i.e. the professional engineer, the use of mathematical knowledge tends to be implicit. Engineers seldom apply the mathematics methods they learnt at university but frequently use the concepts when exploring engineering challenges (Treveyan, 2014). Whitfield (1975) makes the observation that the ‘personality’ of the engineer, when engaged in problem resolution, will be important, particularly the willingness and ability to form hypotheses and to tolerate uncertainty and risk. These attributes coupled with the ability to apply learned knowledge and skills enables the engineer to resolve the problem creatively. To expedite the process of moving from procedural knowledge to efficient and effective problem resolution and hence being able to employ the higher order thinking skills, engineering mathematics needs to foster the ability of students to explore a challenge and apply the knowledge and skills they have acquired and have the technical lexicon to discuss their resolutions. A word of caution is necessary when attempting a problem-based approach to the learning and teaching of engineering mathematics. In some cases, the pre-university learning culture of students is ignored. It should be born in mind that the students will come from a highly structured environment within which they are told what to learn, how to learn and when to learn. Genuine problem resolution is something very few, if any, have encountered before. These novice problem solvers, according to Sweller (1998) tend to employ a ‘means-end analysis’ approach (the approach often taught to students). In other words, they recognise the end goal (the solution) and use techniques to reduce the distance from the initial state (unsolved problem) to the end state by subdividing the problem into a series of sub-goals using problem-solving operators. This approach involves determining the end state, working backwards to identify sub-goals, identify operators and operations to be performed and then start from the initial state to solve the problem by working forwards towards the end state. Although this approach provides a ‘safer’ route to problem-solving (fewer dead ends and blind alleys) it does result in high cognitive loading. Sweller (1988) suggests that the processes involved at a cognitive level are: the problem solver must simultaneously consider the current problem state, the end state, the intermediate goal states and the required procedures to reach intermediate goal states. In order to handle this amount of information, a huge amount of cognitive resources have to be used (particularly memory, recall of mathematical laws and procedures) which can result in the ability to learn schemas for problem-solving to be very limited due to lack of cognitive resources. This situation is further exasperated by the way in which novice problem solvers categorise problems; they tend to group according to the surface structures (Chi & Rees, 1982). In contrast to this, the expert problem solver is able to 'work forward' relying upon acquired schemas built upon the classification of previously solved problems based upon their solution mode.

The development of problem resolution schemas invariably means the student, at some point, will encounter an impasse ie. a point in the process which is unresolvable given their current knowledge and skills. These critical points can result in various outcomes. The student can, by means of trial and improvement resolve the impasse, seek help from those considered to be more knowledgeable, decide the current approach is inappropriate and start again or remain 'stuck' and become frustrated and ultimately disengage from the activity. This sort of behaviour is indicative of cognitive disequilibrium as described by D'Mello and Graeser (2010)

In order to employ the higher order thinking skills and hence become competent problem solvers, students must be given the opportunity to develop schemas which enable them to efficiently, in terms of cognitive resources, resolve problems. The synthesis aspect of HOTS can be directly related to schema acquisition since, by definition, synthesis is the process of combining different elements to form a connected whole (related to the classification of problems). Analysis is the process of examining the problem in an organised way and evaluation is the process of determining the quality, effectiveness and efficiency of the problem resolution.

AIM AND OBJECTIVES

The question this paper seeks to address is: can the process of developing higher order thinking skills based on the acquisition of mathematical competencies be initiated within a first-year engineering undergraduate programme using mathematics as a vehicle? This question can be subdivided into:

(a) Is there a process in which explicit knowledge can become implicit?
(b) Can students learn to form realistic and sensible hypotheses?
(c) Can students learn to tolerate uncertainty and take risks?
(d) Can students learn to reflect upon there resolutions to offer a sensible solution?

METHODOLOGICAL APPROACH

At the end of the programme of study, the students were asked to complete a questionnaire about their experience of this approach to learning engineering mathematics. The questionnaire comprised of 25 questions broken down into sections on teamwork, problem resolution and learning mathematics. The questionnaire responses (a 5 point Likert scale) were analysed using IBM SPSS ver23 utilising a frequency of response analysis. Each week the investigator would keep field notes based on the questions asked by the students and on the outcome of discussions with each of the teams. Incomplete questionnaires were discarded. Ethical approval was sought and given for this investigation. 182 questionnaires were returned out of the 340 which had been issued (54% return rate). This mixed method provided the investigator with both quantitative and qualitative data which enabled him to cross reference the responses given in the questionnaire with the comments made by the students and with the observations. In this way, it was possible to achieve a realistic level of certainty regarding the validity of the data collected. This methodology was also based on work previously undertaken by the author (Peters, 2015; Peters, 2017)

KEY FINDINGS

Analysis of the questionnaire revealed that 68% of the students thought this approach improved their appreciation of the role of mathematics within engineering, 60% thought it made mathematics more interesting, and 77% thought it improved their mathematics knowledge and skills. Many students described their experience as ‘challenging’, ‘rewarding’, ‘enjoyable’, ‘hard but fun’, ‘hard at first’ and ‘interesting’. Most of the students recognised that the skills they started to develop would help them on their journey to becoming professional engineers.

The findings also highlighted that the majority of the students found it very difficult to make assumptions. They did not like the fact that they would have to find a starting point to begin the problem resolution process. Nor did they like the uncertainty around finding for themselves the mathematics they would need in order to resolve their chosen problem. They also found it difficult to cope with the notion of a problem being ill-formed with multiple potential answers. The idea of ‘taking risks’ and not relying upon a tutor to tell them what to do, again, was very challenging for many of them.

The students’ resolutions to the following challenge illustrated some of the above observations.

Water Supply challenge:

> *‘You have decided that you have had enough of living the ‘rat race’ culture prevalent in the UK. You have done some investigative work and decided to move to Northern Belize, buy some land and build a new life where you are in control. One of the first tasks, after building a shelter, is to build a water storage tank so you can have fresh water all year round.’*

The teams that chose this particular challenge soon discovered that they could use Torricelli’s theorem to answer the initial tasks. When it came to them having to decide upon the initial conditions and the requirements they needed to take into account when designing their model, they began to struggle. For example, deciding upon an appropriate flow rate, deciding upon the amount of water they would require in addition to that required for drinking, how to collect the water and how to transport it to their dwelling.

The discussions with the students also revealed that they found it challenging when an impasse was reached in their investigation. In some cases, the impasse was resolved through group discussions, some groups would discard the resolution path they were following and restart the whole problem resolution process and others would ‘give up’ and seek help from a member of staff.

A subsequent cohort was asked about their experiences of working as a part of the team, problem resolution and how they thought this class helped them develop as professional engineers.

Team working.

The students were asked to describe their experiences of working in a team. In response to the question ‘How would you describe your experience as working as part of a team?’ the following statements were made: *it gave me insights into leadership and group dynamics, helped revise my maths knowledge, improved my social skills, helped me make connections, helped me to learn how to delegate, developed my logical thinking, taught me the difficulty in dealing with non-committed team members, built my confidence in speaking in public, developed my organisational skills and taught me to look at challenges in different ways.*

Development as a professional engineer (with regards to team working).

In response to ‘How do you think this experience will have helped your development as a professional engineer?’ responses included: *not much, sharing of ideas, empathy with others, accepting a consensus, learning to compromise, learning to cope with adversity, time management, improved organisational skills, importance of cooperation, childish and not useful, not to think people have the same mentality as yourself, made me feel better prepared* *for the future, learned to be creative and innovative, develop patience and improved my research skills.*

Problem resolution

In response to 'How would you describe your experience of problem resolution?' answers included*: challenging, useful, value of research, tough, presenting results difficult, enhanced communication skills, rewarding, excellent, found it challenging to make assumptions, developed my critical thinking skills, found it difficult on deciding on appropriate maths to use, finding a starting point was difficult, fulfilling, quite an adventure, helped me develop trial and error technique, makes you really think and makes you think outside the box.*
Development as a professional engineer (with regards to problem resolution).
When asked 'How do you think this experience will have helped your development as a professional engineer?' responses included: *not very much, taught me to seek help when needed, to work in a team effectively, project management particularly the allocation of resources, maintained my interest in engineering, in gave me the opportunity to put maths into context, it allowed me to apply my knowledge to real world problems, it gave me something to speak about in interviews, help me develop thinking methods to overcome challenges, it taught me perseverance, it provided preparation for the workplace, helped me to develop initiative and taught me how to learn from my mistakes.*

DISCUSSION

Although from a preliminary reading it may appear that this approach does not reflect much of a change from a traditional engineering mathematics curriculum. One of the reasons the module was designed in this way was to ease the transition from the highly ordered learning environment of a school or college to one where the learner was expected to take more responsibility for their own learning. It was decided that if a purely problem-based approach to learning engineering mathematics was adopted then many of the students would be overwhelmed, with the net contrary result of inadvertently putting in place a substantial barrier to learning. The different elements of the module were designed with specific purposes in mind. The traditional lectures and tutorials were elements the students would expect from a university programme and thus feel they were being 'taught'. They also provided a means of the students enhancing their mathematical knowledge and skills. The guided, self-learning of Matlab afforded the students the opportunity to begin the journey of becoming autonomous learners. In semester two the students would then have a reasonable probability of successfully resolving an open-ended, ill-formed problem. Another reason this format was adopted was to focus on what was meant to be learned without imposing a huge cognitive load. In other words, once the students were confronted with their chosen challenge, they should have had various schemas in place, albeit in an embryonic form, so they would be able to focus and direct their cognitive resources to find a sensible, efficient and valid resolution to their challenge. The challenges themselves could be resolved with fundamental mathematics but also higher level mathematics could have been used if the teams were confident and competent in such procedures. The emphasis was very much upon the teams to decide the mathematical 'tools' they needed to resolve the challenge.
Although many of the mathematical competencies listed in the SEFI document are not explicitly recognised by the students, many of them are implicit in the process of developing a Matlab model to resolve a problem. Due to the nature in which the class was run, students had to think and reason mathematically without relying on a tutor to provide them with the 'answers' if they encountered difficulties. Many students found this extremely challenging to start with and asked questions like 'what equations do I need?', 'how do I code this in Matlab?', 'Is this assumption right?' Staff were encouraged not to intervene and encourage students to adopt the principles of problem resolution as detailed in the information they were given during the delivery of the programme. At the start, the students found this response difficult to deal

with since making assumptions, being allowed to make mistakes and having to find information for themselves was not something they were used to. After a few weeks, the students realised that the staff would guide them but not answer specific questions.

The students were also informed to use the problem worksheets as a guide and encouraged to develop the problem by posing 'what if?' type scenario questions in order to extend the problem to reflect the realities of providing a sensible resolution. Inherent in the problem resolution process is the need for the students to represent physical entities mathematically using conventional notation. This invariably involves them in handling mathematical symbolism and having to interpret the equations and expressions in order to apply them to their particular challenge.

Team working

It was apparent, from observations, the more successful teams set up a means of communicating outside of the scheduled class. They utilised such technologies as on-line chat systems and arranged meetings to discuss their projects. These teams also engaged in meaningful discussions about their challenge and in the main, settled on the consensus as to proceeding with their resolution. At times some of these discussions were quite critical and in danger of becoming very heated especially where one of the team members held strong opinions on how to resolve the challenge. There were some personality clashes in some of the teams which resulted in the alienation of certain members. The major issue with non-functional teams was around team members not attending the class or arranged meetings. This led to a lot of frustration for the team members who were committed to resolving the challenge.

Problem resolution

Observation of the teams revealed some of them were confounded by the prospect of not being given the equations necessary to resolve their challenge. Initially, the majority of the teams would ask what equations were required or the Matlab code they needed. The facilitators were informed not to answer these questions directly but to guide the teams in how they approached the resolution of their chosen problem. From a student perspective, especially those who had no or very little experience of genuine problem solving, this was a major challenge. They had become acclimatised to solving problems where there was only one correct answer and metaphorically speaking, using a mathematics machine where you inputted the values for given variables and the answer came out the other end. It seems the notion of trial and improvement was not a procedure they were comfortable with. A key comment was about the learning of perseverance. Too often, from observations, the teams would be ready to give up when they encountered a problem which they perceived as complex or impossible to resolve. This attitude diminished as the class progressed and their confidence and competence in problem resolution increased. This development was confirmed by comments such as 'helped me to develop initiative and taught me how to learn from my mistakes'.

It was also interesting to note the way the teams went about selecting a problem. Most of the teams selected a problem on their perception of how easy it would be to 'solve'. In some instances, this proved to be an erroneous process. For example, one challenge was to design a speed control hump to stop vehicles speeding through a built-up area. On the face of it, the students thought this would be straightforward but once they tried to mathematically model a speed hump they realised that it was not straightforward.

The assessment via a poster was introduced in order to afford the opportunity for students to discuss their work with members of staff. In the first instance, the students were expected to outline the problem, how they went about developing a resolution and how their particular resolution provided a good solution to the challenge. They were then asked specific technical questions about their resolution. Students found this method of assessment more challenging than they first expected but some commented on how the experience built their confidence for future presentations and how it taught them to prioritise information related to their resolution. The consensus from the students can be summarised by a statement made by one of them:

> *‘It was a good idea to allow students to think and build their projects around their own assumptions and thoughts. Allow them to have a better insight on how to manage team and time.’*

The different forms of assessment for this module were designed to assess specific aspects. The summative, terminal examination was designed to test the students’ procedural knowledge, the Matlab model building their conceptual knowledge and the poster their communication skills and ability to work as an effective member of a team.

CONCLUSIONS & RECOMMENDATIONS

There is an inherent danger that current engineering education is producing graduates who can perform well in examinations but in reality, do not possess, what the RAE has termed, the Engineering Habits of Mind (EHoM) or what the CDIO syllabus refers to as Engineering Reasoning and Problem Solving, Experimentation and Knowledge Discovery. The outcome from this educational process is that many graduates are only capable of solving well-formed, closed-problems ie. the type of question given in an examination which only requires the application of a well-defined procedure. This situation, therefore, requires many companies having to invest in the extensive training of graduate engineers in order for them to be useful employees of the organisation.

This study has shown that the majority of students when they first arrive at a university are competent in applying their procedural knowledge of mathematics but when it comes to analysing and resolving a simple engineering problem, they find it extremely challenging to make assumptions, identify and select appropriate mathematical constructs in order to create an abstract model and interpret the outcomes from their model. It has also shown that their ability to ask the ’right’ questions in order to work towards a problem resolution is limited. They also seem to lack fundamental skills in working within a team, communicating technical ideas, prioritising activities in relation to managing their time.

In a purely traditional model of teaching engineering mathematics ie. lecture, tutorial and examinations, the students are rarely given the opportunity to articulate their ideas and discuss mathematics. Most traditional forms of assessment take the form of class tests and examinations which, by their very nature, make it extremely difficult to assess HOTS and do not encourage teamwork or the ability to communicate technical information, particularly mathematics. Unfortunately, this adherence to a ‘traditional’ approach tends to be advocated by the Professional Bodies and many universities who are more concerned with the elimination of opportunities for students to cheat and plagiarise even though evidence (RAE, 2010) from reported conversations with professional engineers suggest that authentic problem solving, teamwork and the ability to communicate ideas are extremely important and more specifically: “industry … regards the ability to apply theoretical knowledge to real industrial problems as the single most desirable attribute in new recruits. …’ (RAE, 2007).

In an ideal world engineering mathematics would be integrated within other technical subjects and hence eliminating the ‘subject silos’ of engineering programmes which are encouraged by a modular based system. A more project-based approach should also go some way to encourage students to become ‘deep’ learners rather than ‘strategic’ ones where they focus on passing modules in the hope of accumulating enough credits to pass the year. The totally project-based approach has numerous challenges, such as accreditation, assessment, moving away from a teacher-centric approach to a student-centred approach and the management of student and staff expectations.

Engineering schools who are actively looking for alternative ways to facilitate the learning of engineering mathematics must be cognizant of the entry profiles of their students. Universities who are considered to be high tariff institutions would probably attract students who are confident and competent enough to cope with a project-based approach whereas institutions at the other end of the scale, or who lower entry criteria for clearing, would have to ensure a robust academic support structure was in place to provide a safety net for students who would struggle in such an environment.

By the end of this module, many students appreciated the opportunity to start to develop the skills required by an engineer in the 21st century. In terms of this investigation, the students started to develop the stated mathematical competencies and hence develop schemata whereby explicit knowledge became implicit. They also developed in confidence and learned to trust their own abilities with the realisation that they may not always be correct. They also began to develop a critical evaluation mindset whereby they were able, as a team, to look at their resolutions and decide whether they were sensible.

In order to continue the development of EHoM this approach of allowing students to tackle ill-formed, open-ended problems should be continued throughout their time at university and incorporated into the other subjects they study. This process will not produce engineers who are competent on graduation to resolve the many complex problems inherent in our modern world but it will give them a firm foundation and mindset on which companies can shape the engineers they require.

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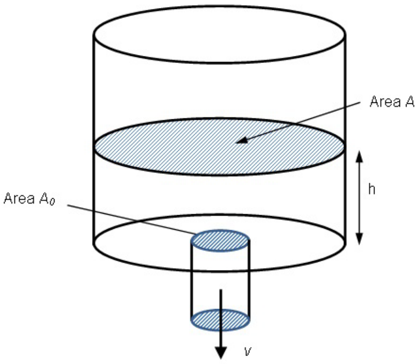
APPENDIX 1

Applying Mathematics - Supplying Water

Scenario

You have decided that you have had enough of living the 'rat race' culture prevalent in the UK. You have done some investigative work and decided to move to Northern Belize, buy some land and build a new life where you are in control. One of the first tasks, after building a shelter, is to build a water storage tank so you can have fresh water all year round. In your investigations you found out that Northern Belize has a rainy season between June and November where, on average, 1524mm of rain falls. You decide upon a cubical tank with a water outlet at the bottom. Your initial 'guess' at the dimensions for your tank were: sides 3m with a drain hole of diameter 0:1m. Unfortunately you can only find information on a cylindrical tank as shown in the diagram.

Initial Tasks



(a) Find a differential equation relating the height, h of the water at a time t. (b) Solve this equation for the initial conditions t = 0,h = 2. (c) How long, in minutes, does it take to empty the tank which is 2m full? (d) Decide how much fresh water you require per year and design an appropriate size tank.

Main Task

Using Matlab develop a mathematical model to investigate different sizes of tanks and different flow rates so you have access to water all year round.

BIOGRAPHICAL INFORMATION

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